Задача 8. Покажите, что последовательность $a_n = (1 + \frac{1}{n})^n$ возрастающая.

a(n+1)/an>1

(a+b)^n=C(n,0)*a^n + C(n,1)*a^(n-1)b^1+C(n,2)*a^(n-2)b^2+...

НОД и НОК НОД(a,b)*НОК(a,b)=a*b НОК(a,b)=a*b/НОД(a,b) Дейкстра одновременно считает НОД по Евклиду и НОК заодно	C(n,k)=? C(1000,1)=1000!/999!*1!=1000

C((n+1),k)1/(n+1)^k
C(n,k)*1/n^k

C((n+1),k)*1/(n+1)^k=(n+1)!/[k!*(n+1-k)!]*1/(n+1)^k=(n+1)*(n+1-1)*(n+1-2)*...*(n+1-(k-1)) / k!*1/(n+1)^k=
=1*(1 - 1/(n+1))*(1 - 2/(n+1))*....*(1-(k-1)/(n+1)) / k!

C(n,k)*1/n^k=n!/[k!*(n-k)!]*1/n^k=(n)*(n-1)*(n-2)*...(n-(k-1))/[k!**1/n^k]=
=1*(1-1/n)*(1-2/n)*...(1-(k-1)/n)/ k!

C((n+1),k)*1/(n+1)^k > C(n,k)*1/n^k

n>0

a(n+1)/an>1

(1+1/(n+1))^(n+1) / (1+1/n)^n =
= ((n+2)/(n+1))^(n+1) / ((n+1)/(n))^n=
= ((n+2)/(n+1))^(n+1) * ((n)/(n+1))^n=
= n^n(n+2)^(n+1)/(n+1)^(2n+1)

a(n+1)=(1+1/(n+1))^(n+1) = 1^(n+1)+C((n+1),1)1/(n+1)*1^(n+1)+C((n+1),2)1/(n+1)^2+...+C(n+1,n)1^1*1/(n+1)^n+C(n+1,n+1)*1/(n+1)^(n+1)   C(n+1,n+1)*1/(n+1)^(n+1)>0
an=(1+1/n)^n = 1^n+C(n,1)*1/n+C(n,2)*1/n^2+...C(n,n)1/n^n

C((n+1),1)= (n+1)!/1!*(n!)=(n+1)!/n!
C((n),1)= n!/(n-1)!

(n+1)!/[(n+1)n!] / n!/[n(n-1)!] = (n+1)!/(n+1)n! / n!/n! = 1
(n+1)!/[(n+1)n!] / n!/[n(n-1)!] =(n+1)!/[(n+1)n!] * [n(n-1)!] / n! = (n+1)!/[(n+1)n!]= n!/n!=1

C((n+1),2)= (n+1)!/[2!*(n-1)!]=(n+1)!/[2!*(n-1)!]=n*(n+1)/2!
C((n),2)= n!/2!*(n-2)!=(n-1)n/2!

n*(n+1)/2! * 1/(n+1)^2 / (n-1)n/2! * 1/n^2 = n*(n+1) / [2! (n+1)^2] / (n-1)n / [2! * n^2] = n*(n+1) / [2! (n+1)^2] * [2! * n^2] / (n-1)n =
= (n+1) / [(n+1)^2] * [n^2] / (n-1) = (n+1)n^2/[(n-1)(n+1)^2]=n^2/[(n-1)(n+1)]=n^2/(n^2-1)>1

(n+1)!/[k!*(n+1-k)!] * 1/(n+1)^k / n!/[k!*(n-k)!] * 1/n^k =
=(n+1)!/[k!*(n+1-k)!] * 1/(n+1)^k *k!*(n-k)!n^k/ n!=
=(n+1)!k!*(n-k)!n^k / [k!*(n+1-k)!]*(n+1)^k*n!=
=(n+1)n!(n-k)!n^k / [(n+1-k)!]*(n+1)^k*n!=
=(n+1)(n-k)!n^k / [(n+1-k)!]*(n+1)^k=
=(n-k)!n^k / [(n+1-k)!]*(n+1)^(k-1)=
=n^k / [(n+1-k)]*(n+1)^(k-1)=
= 1/(n+1-k) * 1/(n+1)^(k-1) / 1/n^k = n^k / [(n+1-k)*(n+1)^(k-1)]=
=n^k / [(n+1)^(k)-k(n+1)^(k-1)]